

積分表

基本積分

1. $\int du = u + C$
2. $\int a du = au + C$
3. $\int [f(u) + g(u)] du = \int f(u) du + \int g(u) du$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$
5. $\int \frac{du}{u} = \ln |u| + C$

含 $a + bu$ 的積分

6. $\int \frac{u du}{a + bu} = \frac{1}{b^2} [a + bu - a \ln |a + bu|] + C$
7. $\int \frac{u^2 du}{a + bu} = \frac{1}{b^3} \left[\frac{1}{2}(a + bu)^2 - 2a(a + bu) + a^2 \ln |a + bu| \right] + C$
8. $\int \frac{u du}{(a + bu)^2} = \frac{1}{b^2} \left[\frac{a}{a + bu} + \ln |a + bu| \right] + C$
9. $\int \frac{u^2 du}{(a + bu)^2} = \frac{1}{b^3} \left[a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right] + C$
10. $\int \frac{u du}{(a + bu)^3} = \frac{1}{b^2} \left[\frac{a}{2(a + bu)^2} - \frac{1}{a + bu} \right] + C$
11. $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$
12. $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
13. $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} + \frac{1}{a^2} \ln \left| \frac{u}{a + bu} \right| + C$

含 $\sqrt{a + bu}$ 的積分

14. $\int u \sqrt{a + bu} du = \frac{2}{15b^3} (3bu - 2a)(a + bu)^{3/2} + C$
15. $\int u^2 \sqrt{a + bu} du = \frac{2}{105b^3} (15b^2u^2 - 12abu + 8a^2)(a + bu)^{3/2} + C$
16. $\int u^n \sqrt{a + bu} du = \frac{2u^n(a + bu)^{3/2}}{b(2n + 3)} - \frac{2an}{b(2n + 3)} \int u^{n-1} \sqrt{a + bu} du$
17. $\int \frac{u du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$
18. $\int \frac{u^2 du}{\sqrt{a + bu}} = \frac{2}{15b^3} (3b^2u^2 - 4abu + 8a^2) \sqrt{a + bu} + C$
19. $\int \frac{u^n du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2an}{b(2n + 1)} \int \frac{u^{n-1} du}{\sqrt{a + bu}}$
20. $\int \frac{du}{u \sqrt{a + bu}} = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, & \text{若 } a > 0 \\ \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, & \text{若 } a < 0 \end{cases}$
21. $\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$
22. $\int \frac{\sqrt{a + bu} du}{u} = 2\sqrt{a + bu} + a \int \frac{du}{u \sqrt{a + bu}}$
23. $\int \frac{\sqrt{a + bu} du}{u^n} = -\frac{(a + bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)} \int \frac{\sqrt{a + bu} du}{u^{n-1}}$

含 $a^2 \pm u^2$ 的積分

24. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
25. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C = \begin{cases} \frac{1}{a} \tanh^{-1} \frac{u}{a} + C, & \text{若 } |u| < a \\ \frac{1}{a} \coth^{-1} \frac{u}{a} + C, & \text{若 } |u| > a \end{cases}$
26. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C = \begin{cases} -\frac{1}{a} \tanh^{-1} \frac{u}{a} + C, & \text{若 } |u| < a \\ -\frac{1}{a} \coth^{-1} \frac{u}{a} + C, & \text{若 } |u| > a \end{cases}$

含 $\sqrt{u^2 \pm a^2}$ 的積分

在公式 27~38 中,

以 $\sinh^{-1} \frac{u}{a}$ 代 $\ln(u + \sqrt{u^2 + a^2})$

以 $\cosh^{-1} \frac{u}{a}$ 代 $\ln|u + \sqrt{u^2 - a^2}|$

以 $\sinh^{-1} \frac{a}{u}$ 代 $\ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right|$

$$27. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$28. \int \sqrt{u^2 \pm a^2} du = \frac{u}{2} \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$29. \int u^2 \sqrt{u^2 \pm a^2} du = \frac{u}{8} (2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - \frac{a^4}{8} \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$30. \int \frac{\sqrt{u^2 + a^2} du}{u} = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$31. \int \frac{\sqrt{u^2 - a^2} du}{u} = \sqrt{u^2 - a^2} - a \sec^{-1} \left| \frac{u}{a} \right| + C$$

$$32. \int \frac{\sqrt{u^2 \pm a^2} du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$33. \int \frac{u^2 du}{\sqrt{u^2 \pm a^2}} = \frac{u}{2} \sqrt{u^2 \pm a^2} - \frac{\pm a^2}{2} \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$34. \int \frac{du}{u \sqrt{u^2 + a^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$35. \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

$$36. \int \frac{du}{u^2 \sqrt{u^2 \pm a^2}} = -\frac{\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C$$

$$37. \int (u^2 \pm a^2)^{3/2} du = \frac{u}{8} (2u^2 \pm 5a^2) \sqrt{u^2 \pm a^2} + \frac{3a^4}{8} \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$38. \int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{u}{\pm a^2 \sqrt{u^2 \pm a^2}} + C$$

含 $\sqrt{a^2 - u^2}$ 的積分

$$39. \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$40. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$41. \int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$42. \int \frac{\sqrt{a^2 - u^2} du}{u} = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C = \sqrt{a^2 - u^2} - a \cosh^{-1} \frac{a}{u} + C$$

$$43. \int \frac{\sqrt{a^2 - u^2} du}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \sin^{-1} \frac{u}{a} + C$$

$$44. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$45. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C = -\frac{1}{a} \cosh^{-1} \frac{a}{u} + C$$

$$46. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$47. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$48. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

含 $2au - u^2$ 的積分

$$\begin{aligned}
 49. \int \sqrt{2au - u^2} du &= \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(1 - \frac{u}{a} \right) + C & 54. \int \frac{u du}{\sqrt{2au - u^2}} &= -\sqrt{2au - u^2} + a \cos^{-1} \left(1 - \frac{u}{a} \right) + C \\
 50. \int u \sqrt{2au - u^2} du &= \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} & 55. \int \frac{u^2 du}{\sqrt{2au - u^2}} &= -\frac{(u+3a)}{2} \sqrt{2au - u^2} \\
 &+ \frac{a^3}{2} \cos^{-1} \left(1 - \frac{u}{a} \right) + C & &+ \frac{3a^2}{2} \cos^{-1} \left(1 - \frac{u}{a} \right) + C \\
 51. \int \frac{\sqrt{2au - u^2} du}{u} &= \sqrt{2au - u^2} + a \cos^{-1} \left(1 - \frac{u}{a} \right) + C & 56. \int \frac{du}{u \sqrt{2au - u^2}} &= -\frac{\sqrt{2au - u^2}}{au} + C \\
 52. \int \frac{\sqrt{2au - u^2} du}{u^2} &= -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(1 - \frac{u}{a} \right) + C & 57. \int \frac{du}{(2au - u^2)^{3/2}} &= \frac{u-a}{a^2 \sqrt{2au - u^2}} + C \\
 53. \int \frac{du}{\sqrt{2au - u^2}} &= \cos^{-1} \left(1 - \frac{u}{a} \right) + C & 58. \int \frac{u du}{(2au - u^2)^{3/2}} &= \frac{u}{a \sqrt{2au - u^2}} + C
 \end{aligned}$$

含三角函數的積分

$$\begin{aligned}
 59. \int \sin u du &= -\cos u + C & 75. \int \tan^n u du &= \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u du \\
 60. \int \cos u du &= \sin u + C & 76. \int \cot^n u du &= -\frac{1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u du \\
 61. \int \tan u du &= \ln |\sec u| + C & 77. \int \sec^n u du &= \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u du \\
 62. \int \cot u du &= \ln |\sin u| + C & 78. \int \csc^n u du &= -\frac{1}{n-1} \csc^{n-2} u \cot u + \frac{n-2}{n-1} \int \csc^{n-2} u du \\
 63. \int \sec u du &= \ln |\sec u + \tan u| + C & 79. \int \sin mu \sin nu du &= -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C \\
 &= \ln \left| \tan \left(\frac{1}{2} \pi + \frac{1}{2} u \right) \right| + C & 80. \int \cos mu \cos nu du &= \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C \\
 64. \int \csc u du &= \ln |\csc u - \cot u| + C & 81. \int \sin mu \cos nu du &= -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C \\
 &= \ln \left| \tan \frac{1}{2} u \right| + C & 82. \int u \sin u du &= \sin u - u \cos u + C \\
 65. \int \sec^2 u du &= \tan u + C & 83. \int u \cos u du &= \cos u + u \sin u + C \\
 66. \int \csc^2 u du &= -\cot u + C & 84. \int u^2 \sin u du &= 2u \sin u + (2 - u^2) \cos u + C \\
 67. \int \sec u \tan u du &= \sec u + C & 85. \int u^2 \cos u du &= 2u \cos u + (u^2 - 2) \sin u + C \\
 68. \int \csc u \cot u du &= -\csc u + C & 86. \int u^n \sin u du &= -u^n \cos u + n \int u^{n-1} \cos u du \\
 69. \int \sin^2 u du &= \frac{1}{2} u - \frac{1}{4} \sin 2u + C & 87. \int u^n \cos u du &= u^n \sin u - n \int u^{n-1} \sin u du \\
 70. \int \cos^2 u du &= \frac{1}{2} u + \frac{1}{4} \sin 2u + C & 88. \int \sin^m u \cos^n u du & \\
 71. \int \tan^3 u du &= \tan u - u + C & &= -\frac{\sin^{m-1} u \cos^{n+1} u}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} u \cos^n u du \\
 72. \int \cot^2 u du &= -\cot u - u + C & &= \frac{\sin^{m+1} u \cos^{n-1} u}{m+n} + \frac{n-1}{m+n} \int \sin^m u \cos^{n-2} u du \\
 73. \int \sin^n u du &= -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u du & & \\
 74. \int \cos^n u du &= \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u du & &
 \end{aligned}$$

含反三角函數的積分

$$89. \int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$$

$$90. \int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$91. \int \tan^{-1} u \, du = u \tan^{-1} u - \ln \sqrt{1+u^2} + C$$

$$92. \int \cot^{-1} u \, du = u \cot^{-1} u + \ln \sqrt{1+u^2} + C$$

$$93. \int \sec^{-1} u \, du = u \sec^{-1} u - \ln |u + \sqrt{u^2-1}| + C$$

$$= u \sec^{-1} u - \cosh^{-1} u + C$$

$$94. \int \csc^{-1} u \, du = u \csc^{-1} u + \ln |u + \sqrt{u^2-1}| + C$$

$$= u \csc^{-1} u + \cosh^{-1} u + C$$

含指數函數與對數函數的積分

$$95. \int e^u \, du = e^u + C$$

$$96. \int a^u \, du = \frac{a^u}{\ln a} + C$$

$$97. \int u e^u \, du = e^u(u-1) + C$$

$$98. \int u^n e^u \, du = u^n e^u - n \int u^{n-1} e^u \, du$$

$$99. \int u^n a^u \, du = \frac{u^n a^u}{\ln a} - \frac{n}{\ln a} \int u^{n-1} a^u \, du + C$$

$$100. \int \frac{e^u \, du}{u^n} = -\frac{e^u}{(n-1)u^{n-1}} + \frac{1}{n-1} \int \frac{e^u \, du}{u^{n-1}}$$

$$101. \int \frac{a^u \, du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1} \int \frac{a^u \, du}{u^{n-1}}$$

$$102. \int \ln u \, du = u \ln u - u + C$$

$$103. \int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$104. \int \frac{du}{u \ln u} = \ln |\ln u| + C$$

$$105. \int e^{au} \sin nu \, du = \frac{e^{au}}{a^2 + n^2} (a \sin nu - n \cos nu) + C$$

$$106. \int e^{au} \cos nu \, du = \frac{e^{au}}{a^2 + n^2} (a \cos nu + n \sin nu) + C$$

含雙曲線函數的積分

$$107. \int \sinh u \, du = \cosh u + C$$

$$108. \int \cosh u \, du = \sinh u + C$$

$$109. \int \tanh u \, du = \ln |\cosh u| + C$$

$$110. \int \coth u \, du = \ln |\sinh u| + C$$

$$111. \int \operatorname{sech} u \, du = \tan^{-1}(\sinh u) + C$$

$$112. \int \operatorname{csch} u \, du = \ln |\tanh \frac{1}{2}u| + C$$

$$113. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$114. \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$115. \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$116. \int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

$$117. \int \sinh^2 u \, du = \frac{1}{4} \sinh 2u - \frac{1}{2}u + C$$

$$118. \int \cosh^2 u \, du = \frac{1}{4} \sinh 2u + \frac{1}{2}u + C$$

$$119. \int \tanh^2 u \, du = u - \tanh u + C$$

$$120. \int \coth^2 u \, du = u - \coth u + C$$

$$121. \int u \sinh u \, du = u \cosh u - \sinh u + C$$

$$122. \int u \cosh u \, du = u \sinh u - \cosh u + C$$

$$123. \int e^{au} \sinh nu \, du = \frac{e^{au}}{a^2 - n^2} (a \sinh nu - n \cosh nu) + C$$

$$124. \int e^{au} \cosh nu \, du = \frac{e^{au}}{a^2 - n^2} (a \cosh nu - n \sinh nu) + C$$